Networking academia:
Categories as a cross-disciplinary research language

Executive synopsis:

A network culture is emerging in human society. It takes on a wide variety of forms, from the on-line social networks between individuals, to information networks (e.g. Lokey's Business Wire), to virtual trading floors for financial securities, to global delivery networks of goods and services: everyone is linking up. In order to efficiently study this phenomenon, one must construct rigorous definitions which are general enough to encompass many different types of networks, but precise enough to allow researchers in disparate fields to accurately communicate and compare their findings. Although definitions for various kinds of networks have been made, they are neither rigorous nor broadly applicable, resulting in a lack of good communication between distinct fields. Category theory may be useful in solving this problem. Category theory is a branch of mathematics designed to provide a unifying language for all of mathematics. It has revolutionized mathematical research since its invention in the 1940s and has begun breaking ground in computer science and physics as well, which is evidence of its generality and applicability. I believe that category theory could play an even broader role in academia at large. The goal of this project is to apply category theory to the theory of networks in order to provide a rigorous yet flexible set of rules for specifying and studying networks in a broad array of social and physical settings. I hope to devise a common language for economists, computer scientists, linguists, mathematicians, and researchers in many other fields, which in turn will strengthen the network that is academia itself.

Description:

Background:

At the beginning of the 20th century, the field of mathematics consisted of a loose affiliation of distinct and somewhat disassociated subfields. Group theorists studied symmetries, algebraic geometers studied the solutions to systems of equations, topologists studied shapes in space, analysts studied the behavior of smooth functions. While the techniques from one subfield sometimes carried over to others, researchers in each subfield often spoke their own jargon and proved their own theorems.

In the early 1940s, Samuel Eilenberg and Saunders Mac Lane introduced a new theory and language of mathematics called “category theory.” In philosophy, a category consists of a collection of objects under consideration (e.g. the category of people or the category of metaphors) and the relationships among these objects. In mathematics, a category is a collection of objects and a collection of arrows from object to object, each of which represents some form of relationship. The arrows must satisfy certain axioms that we will not detail here, but these axioms are general enough that categories are ubiquitous in mathematics while being strict enough to ensure that the concept is precise and powerful. Unlike the philosophical notion of category, the mathematical notion is such that one can find formal relationships between totally different categories, for example between the category of shapes in 20-dimensional space and the category of finite sets.

One generally uses category theory to define and organize layers of abstraction. In so doing, one tends to uncover new and useful structures within well-studied fields. The process of simply fitting an object of study into the categorical framework has led to many discoveries in mathematics that had gone unnoticed for decades.

Category theory has unified much of mathematics. It allows researchers in every branch of mathematics to converse in a common language and to transport ideas and theorems between disparate fields. Most papers written today use the language of categories to describe results. Category theory is becoming an important research tool in theoretical computer science and physics as well, a testament to its ability to cross disciplinary boundaries.

Currently the social sciences are suffering from a lack of interdisciplinary communication just as math was many years ago. While researchers in different fields may be able to converse, they do not use the same language and are not capable of readily transporting ideas and results from one field to another. However, a common idea seems to be emerging: the network.

Networks are ubiquitous among the subjects studied in academia. In economics, one has the market, which is little more than a network of individuals communicating their conceptions of value. In sociology, one could almost define a culture by the communication network of individuals that compose it. In biology, one studies many different networks, including systems of nerve connections called neural networks, in which some basic form of information transfer leads to all that is knowledge and ability. In linguistics, one notes that no word in the dictionary is defined on its own and that it is only the network of connections between different words that endows the language with the ability to convey meaning.

A network consists of a set of nodes (symbolizing people, computers, neurons, etc.) and a set of connections between them (symbolizing conversations, emails, synapses, etc.). A category consists of a set of nodes and a set of arrows between them. Clearly, there is much similarity between networks and categories. However, to really capture the essence of a network, one must choose exactly the right category to represent it.

Roughly, we have the following idea in mind. At each node of the network, we attach a database representing “what the node knows” (or more precisely, its semantic primitives). To every edge between two nodes, we attach a database representing “what these two nodes can discuss together” (more precisely, the semantic primitives that they have in common.) For example, a pair of siblings can readily discuss their family history, whereas with a complete stranger they may only be able to discuss trivialities such as the weather. Although not obvious from the above description, these ideas lend themselves to a categorical formulation of a network that does not seem to have been discussed in the literature. We will apply the tools and theorems of category theory to study global phenomena of these networks.

It is commonly held that a financial market “learns” and processes information, and that it can do so more efficiently than any of the individual investors in that market. The network of neurons in a human brain also learns and processes information in ways of which no individual neuron is even remotely capable. So far, this analogy between the market and the brain is just an analogy; however, it may be that by formulating networks in terms of categories, one could see precisely how the learning phenomenon emerges in both cases. Furthermore, one may be able to transport findings from biology to economics and back, with mathematical precision.

Project summary:

In some sense, the overarching goal of my proposed project is to help expand the term “applied mathematics.” Currently, applied mathematicians generally use a set of particular tools to solve particular real-world problems. I want to use the overarching structure of mathematics (in the form of category theory) to study overarching structures in the social sciences, as well as structures within individual subjects. In other words, I believe that category theory, although considered by many to be the most abstract form of mathematics, may be general enough to fit into a diverse array of non-mathematical fields and hence become part of the “applied mathematics” gamut. Although I am not the first to think of such a program, the number of people actively working to bring category theory to non-mathematical disciplines is far too small.

To begin the project, I propose to concentrate on applying category theory to networks, which are emerging as important objects of study in a wide variety of academic subjects. One aspect of the project is to understand networks from a mathematical point of view, in as broad yet precise a way as possible. Another is to examine non-mathematical disciplines through the lens of networks and more generally though that of category theory. We will look for ways that category theory might be useful to researchers in other fields. We aim to make our findings comprehensible to as many non-mathematicians as possible, in hopes of strengthening the lines of communication among our respective subjects.

As a category theorist, I was inspired to study networks by reading Mark C. Taylor's book *The Moment of Complexity: Our Emerging Network Culture*. I believe that visiting Taylor at Columbia University would be a good way to begin this project. Similarly, the Santa Fe Institute has many researchers who study networks and communication, such as Jennifer Dunne and Cristopher Moore, from whom I would benefit from speaking with at length. I also plan to visit linguists, economists, biologists, and other researchers at the University of Oregon to open the lines of communication between our fields and determine whether category theory can lead to new connections and ideas. It may be useful to hire a graduate student in one or more of these departments to help with background and other research.

I believe that the project can be completed within 12 to 18 months.

Although the first step is to study networks, it is certainly conceivable that several other projects will emerge tangentially, as we speak to researchers in a broad spectrum of academia, which will provide opportunities for further research.

Rationale:

Epistemology, the study of knowing, is precisely what this project is about. It seems that knowledge is tightly linked to communication, and that communication is tightly linked to networks of individuals. Whether they be social, biological, economic, or physical, networks are responsible for a surprising amount of the phenomena we see around us.

As the world links up in every conceivable way, networks are becoming the new paradigm. Studying how information is transferred in a network (i.e. how financial securities are valued, how words obtain their meaning, how news is delivered and interpreted) is one of the most fascinating subjects of our time. Of course, information transfer in a computer network is very well understood; our hope is to find a foundation that will serve the larger academic community.

Moreover, this project is about improving the quality of communication among research disciplines by devising a single category-theoretic model that will work for all of them. In so doing, the proposed project crosses conventional disciplinary boundaries between mathematics and social sciences. It would foster interdisciplinary and collaborative projects between researchers in a variety of fields.

Most likely, the project will result in future research opportunities as well. Any time new modes of communication are established, interested parties will find much to talk about. We cannot know in advance what these topics may include, but complexity theory, epistemology, and cellular automata theory are certainly candidates for further research.

Personnel:

David Spivak (primary researcher).
Possibly others, not yet determined, including graduate students from non-mathematical disciplines.

Budget:

Travel (New York, Santa Fe, other): $3,000.
Books and research materials: $500.
Summer 2009 support: One month's salary ($4,200).
Graduate Student support: $0 – 5,000.

Total: $7,700 – 12,700.